# Infrared divergences of $B$ meson exclusive decays to P-wave charmonia in QCD factorization and nonrelativistic QCD

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In the framework of QCD factorization, we study the $B$ meson exclusive decays $B \to \chi_{cJ} K$ where the spin-triplet P-wave charmonium states $\chi_{cJ}(J = 0, 1, 2)$ are described by the color-singlet nonrelativistic wave functions. We find that for these decays (except $\chi_{c1}$) there are infrared divergences arising from nonfactorizable vertex corrections as well as logarithmic end-point singularities arising from nonfactorizable spectator interactions at leading-twist order. The infrared divergences due to vertex corrections will explicitly break down QCD factorization within the color-singlet model for charmonia. Unlike in the inclusive decays where the higher Fock states with color-octet $c\bar{c}$ pair and soft gluon can make contributions to remove the infrared divergences, their contributions can not be accommodated in the exclusive two body decays. As a result, the infrared divergences encountered in exclusive processes involving charmonia may raise a new question to the QCD factorization and NRQCD factorization in B exclusive decays.

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$B$ meson decays, especially exclusive nonleptonic decays, provide an opportunity to determine the parameters of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, to explore CP violation and to observe new physics effects. However, although the underlying weak decay of the $b$ quark is simple, quantitative understanding of nonleptonic $B$-meson decays is difficult due to the complicated strong-interaction effects.

Exclusive $B$-meson decays to final states containing charmonia are especially important due to three reasons. First, those decays e.g. $B \to J/\psi K$ are regarded as the golden channels for CP studies due to their clean experimental signatures and straightforward theoretical interpretations. Secondly, those decays may provide useful information towards the understanding of color suppressed decays. Thirdly, those decays involve two energy scales, the beauty quark mass $m_b$ and charm quark mass $m_c$, and therefore are more subtle in theoretical studies. Since experimentally CLEO, BaBar and Belle Collaborations have provided many measurements on the subject of $B$ exclusive decays to charmonia [1] such as $J/\psi, \psi', \eta_c, \eta_c', \chi_{c0}$, and $\chi_{c1}$, theoretical studies on these issues become necessary.

We will start with the QCD factorization approach proposed by Beneke et al.[2]. It is argued that because the size of charmonium is small($\sim 1/\alpha_s m_\psi$) and its overlap with the $(B, K)$ system is negligible, the same QCD factorization method as for $B \to \pi\pi$ can be used for $B \to J/\psi K$ decay. This small size argument for the applicability of QCD factorization to $B$ exclusive decays to charmonia needs to be tested for specific decay channels. Indeed, explicit calculations for $B \to J/\psi K$ within the QCD factorization approach[3] showed that the nonfactorizable vertex contribution is infrared safe and the spectator contribution is perturbatively calculable at twist-2 order though the theoretical branching ratio is much smaller than the experimental data. Studies on $B \to \eta_c K$ decay[4] also confirmed the qualitative applicability of the small size argument while the quantitative underestimates compared with data were very similar to that encountered in $B \to J/\psi K$. However, further studies on $B \to \chi_{c0} K$ [5] challenged the applicability of QCD factorization for charmonia because of the appearance of non-vanishing infrared divergences in this decay.

In previous studies for exclusive $B$ decays to charmonia within the QCD factorization approach, the light-cone wave function description of the charmonia was adopted. This is fairly appropriate for the S-wave charmonia since the relative momentum $q$ between charmed and anti-charmed quarks can be neglected in the lowest order approximation. However, for P-wave charmonia, $q$ can not be neglected even in the leading-order. Although the light-cone wave function description of charmonia can provide us with some essential features of the problems involved in these decays, it can not give a complete analysis. The more appropriate method is to study charmonia within the non-relativistic bound-state picture or equivalently within the nonrelativistic QCD (NRQCD) framework[6]. In this letter, we report a complete analysis of the infrared unsafety encountered in $B$ exclusive decays to P-wave charmonium states, including $\chi_{c2}$, which was not involved in previous studies, by using the color-singlet non-relativistic wave functions to describe the charmonia.

To proceed, we first give a brief review of the QCD factorization approach. The general idea of the QCD factorization is that in the heavy quark limit $m_b \gg \Lambda_{\rm QCD}$, the

transition matrix elements of operators in the hadronic decay $B \to M_1 M_2$ assumes a simple form at the leading order in $1/m_b$[2]:

$$\langle M_1 M_2|\mathcal{O}_i|B\rangle = F^{BM_1}(m_2^2)\int_0^1 dx\, T^I(x)\phi_{M_2}(x) + \int_0^1 d\xi\, dx\, dy\, T^{II}(\xi,x,y)\phi_B(\xi)\phi_{M_1}(y)\phi_{M_2}(x), \quad (1)$$

where $M_1$ is the recoiled meson and $M_2$ is the emitted meson which is a light meson or a quarkonium. $F^{BM_1}$ is the $B \to M_1$ transition form factor, $\phi_M$ is the light-cone distribution amplitude and $T^{I,II}$ are perturbatively calculable hard scattering kernels. If we neglect strong interaction corrections, formula.(1) reproduces the result of naive factorization. However, hard gluon exchange between $M_2$ and $BM_1$ system implies a nontrivial convolution of hard scattering kernels $T^{I,II}$ with the distribution amplitude $\phi_{M_2}$.

In the non-relativistic bound-state picture, instead of the light-cone amplitude distribution, charmonium can be described by the color-singlet non-relativistic wave function (the role of the higher Fock states with color-octet $c\bar{c}$ pair will be considered later on). Let $p_\mu$ be the total 4-momentum of the charmonium and $q_\mu$ be the relative 4-momentum between $c$ and $\bar{c}$ quarks. For P-wave charmonium, because the wave function at the origin $\mathcal{R}_1(0)$=0, which corresponds to the zeroth order in $q$, we must expand the amplitude to first order in $q$. Thus we have (see, e.g., [7])

$$\mathcal{M}(B \to {}^{2S+1}P_J(c\bar{c})) = \sum_{L_z,S_z}\langle 1L_z;SS_z|JJ_z\rangle\int\frac{\mathrm{d}^4q}{(2\pi)^3}q_\alpha \times\delta(q^0-\frac{|\vec{q}|^2}{M})\psi^\star_{1M}(q)\mathrm{Tr}[\mathcal{O}^\alpha(0)P_{SS_z}(p,0)+\mathcal{O}(0)P^\alpha_{SS_z}(p,0)], \quad (2)$$

where $\mathcal{O}(q)$ represents the rest of the decay matrix element and the spin projection operators $P_{SS_z}(p,q)$ which is constructed in terms of quark and anti-quark spinors as

$$P_{SS_z}(p,q) = \sqrt{\frac{3}{m}}\sum_{s_1,s_2}v(\frac{p}{2}-q,s_2)\bar{u}(\frac{p}{2}+q,s_1)\langle s_1;s_2|SS_z\rangle, \quad (3)$$

and

$$\mathcal{O}^\alpha(0) = \frac{\partial\mathcal{O}(q)}{\partial q_\alpha}|_{q=0}, \quad (4)$$

$$P^\alpha_{SS_z}(p,0) = \frac{\partial P_{SS_z}(p,q)}{\partial q_\alpha}|_{q=0}. \quad (5)$$

After $q^0$ is integrated out, the integral in Eq.(2) is proportional to the derivative of the P-wave wave function at the origin by

$$\int\frac{\mathrm{d}^3q}{(2\pi)^3}q^\alpha\psi^*_{1M}(q) = i\varepsilon^{*\alpha}(L_z)\sqrt{\frac{3}{4\pi}}\mathcal{R}_1^{'}(0), \quad (6)$$

where $\varepsilon^\alpha(L_z)$ is the polarization vector of an angular momentum one system and the value of $\mathcal{R}_1^{'}(0)$ for charmonia can be found in e.g. Ref.[8]

The spin projection operators $P_{SS_z}(p,0)$ and $P^\alpha_{SS_z}(p,0)$ can be written as[7]

$$P_{1S_z}(p,0) = \sqrt{\frac{3}{4M}}\not{\epsilon}^*(S_z)(\not{p}+M), \quad (7)$$

$$P^\alpha_{1S_z}(p,0) = \sqrt{\frac{3}{4M^3}}[\not{\epsilon}^*(S_z)(\not{p}+M)\gamma^\alpha+\gamma^\alpha\not{\epsilon}^*(S_z)(\not{p}+M)]$$

where we have made use of the non-relativistic approximation for the charmonium mass $M \simeq 2m$. Here $m$ is the charmed quark mass.

In the calculation we need following polarization relations for the ${}^3P_J$ states

$$\begin{aligned}
\sum_{L_Z S_Z}\epsilon^{*\alpha}(L_z)\epsilon^{*\beta}(S_z)\langle 1L_z;1S_z|00\rangle &= \frac{1}{\sqrt{3}}(-g^{\alpha\beta}+\frac{p^\alpha p^\beta}{M^2}), \\
\sum_{L_Z S_Z}\epsilon^{*\alpha}(L_z)\epsilon^{*\beta}(S_z)\langle 1L_z;1S_z|1J_z\rangle &= \frac{-i\epsilon^{\alpha\beta\lambda\kappa}p_\kappa\epsilon^*_\lambda(J_z)}{\sqrt{2}M}, \\
\sum_{L_Z S_Z}\epsilon^{*\alpha}(L_z)\epsilon^{*\beta}(S_z)\langle 1L_z;1S_z|2J_z\rangle &= \epsilon^{*\alpha\beta}(J_z), \quad (8)
\end{aligned}$$

where $\epsilon_\lambda(J_z)$ is the usual spin-1 polarization vector and the polarization tensor $\epsilon^{\alpha\beta}(J_z)$ is that appropriate for a spin-2 system which is symmetric under the exchange $\alpha \leftrightarrow \beta$.

The effective Hamiltonian relevant for $B \to \chi_{cJ}K$ is written as[9]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}}\Big(V_{cb}V^*_{cs}(C_1\mathcal{O}_1+C_2\mathcal{O}_2) - V_{tb}V^*_{ts}\sum_{i=3}^{6}C_i\mathcal{O}_i\Big), \quad (9)$$

where $G_F$ is the Fermi constant, $C_i$ are the Wilson coefficients and $V_{q_1q_2}$ are the CKM matrix elements. We do not include the effects of the electroweak penguin operators since they are numerically small. Here the relevant operators $\mathcal{O}_i$ are given by

$$\begin{aligned}
\mathcal{O}_1 &= (\overline{s}_\alpha b_\beta)_{V-A}\cdot(\overline{c}_\beta c_\alpha)_{V-A}, \\
\mathcal{O}_2 &= (\overline{s}_\alpha b_\alpha)_{V-A}\cdot(\overline{c}_\beta c_\beta)_{V-A}, \\
\mathcal{O}_{3(5)} &= (\overline{s}_\alpha b_\alpha)_{V-A}\cdot\sum_q(\overline{q}_\beta q_\beta)_{V-A(V+A)}, \quad (10) \\
\mathcal{O}_{4(6)} &= (\overline{s}_\alpha b_\beta)_{V-A}\cdot\sum_q(\overline{q}_\beta q_\alpha)_{V-A(V+A)},
\end{aligned}$$

where $\alpha$, $\beta$ are color indices and the sum over $q$ runs over $u,d,s,c$ and $b$. Here $(\bar{q}_1q_2)_{V\pm A} = \bar{q}_1\gamma_\mu(1\pm\gamma_5)q_2$.

According to [2] all nonfactorizable corrections are due to Fig.1, and other corrections are factorized into the physical form factors and meson wave functions. Taking nonfactorizable corrections in Fig.1 into account, the decay amplitude for $B \to \chi_{cJ}K(J=0,2)$ in QCD factorization is written compactly as

$$i\mathcal{M} = \frac{G_F}{\sqrt{2}}\Big[V_{cb}V^*_{cs}C_1 - V_{tb}V^*_{ts}(C_4+C_6)\Big]\times A, \quad (11)$$

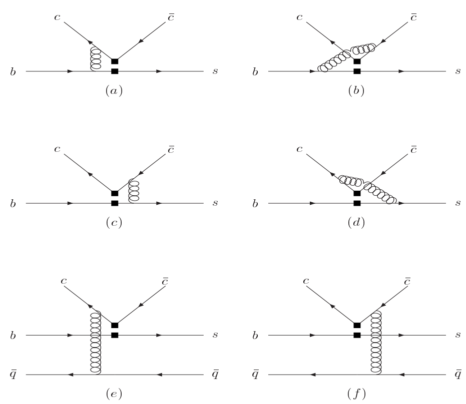


FIG. 1: Feynman diagrams for vertex and spectator corrections to $B \to \chi_{cJ} K$.

where the coefficients $A$ are given by

$$A = \frac{i6\mathcal{R}_1'}{\sqrt{\pi M}} \cdot \frac{\alpha_s}{4\pi}\frac{C_F}{N_c}\Big(F_1 \cdot f_I + \frac{4\pi^2 f_K f_B}{N_c} \cdot f_{II}\Big). \quad (12)$$

Here $N_c$ is the number of colors, $C_F = (N_c^2 - 1)/(2N_c)$, and $F_1$ is the $B \to K$ form factor (see Eq.(13) below). The function $f_I$ is calculated from the four vertex correction diagrams (a, b, c, d) and $f_{II}$ is calculated from the two spectator correction diagrams (e, f) in Fig.1.

The form factors for $B \to K$ are given as

$$\begin{aligned}&\langle K(p_K)|\bar{s}\gamma_\mu b|B(p_B)\rangle = \quad (13)\\ &F_1(p^2)(p_B+p_K)_\mu + [F_0(p^2) - F_1(p^2)]\frac{m_B^2 - m_K^2}{p^2}p_\mu,\end{aligned}$$

where $p = p_B - p_K$ is the momentum of charmonium with $p^2 = M^2$, and $m_B, m_K$ are respectively the masses of $B$, $K$ mesons. We will neglect the kaon mass for simplicity. We can use the ratio between these two form factors as $F_0(p^2)/F_1(p^2) = 1 - p^2/m_B^2$[3]. So we need only one of the two form factors, say, $F_1$ to describe the decay amplitude just like that in Eq.(12).

Since our main task in this letter is to explore the infrared unsafety due to the vertex corrections, we only give the explicit results below for the infrared divergence terms in $f_I$ which are regularized within the gluon mass scheme. We also present the leading-twist results for $f_{II}$. Here $f_I$ and $f_{II}$ correspond respectively to the $T^I, T^{II}$ terms in Eq.(1).

For $^3P_1$ charmonium state $\chi_{c1}$, there is no infrared divergence in $B \to \chi_{c1} K$ decay, and we will give the reasoning shortly (results of this decay within the light-cone wave function description can be found in Ref.[5]).

For $^3P_0$ charmonium state $\chi_{c0}$,

$$f_I = \frac{8 m_b z(1-z+\ln z)}{(1-z)\sqrt{3z}}\ln\big(\frac{\lambda^2}{m_b^2}\big) + finite\ terms, \quad (14)$$

$$\begin{aligned}f_{II} = &\frac{2}{m_b(1-z)\sqrt{3z}}\int_0^1 d\xi \frac{\phi_B(\xi)}{\xi}\int_0^1 dy \frac{\phi_K(y)}{\bar{y}^2}\\ &\times[-2z + (1-z)\bar{y}]. \quad (15)\end{aligned}$$

For $^3P_2$ charmonium state $\chi_{c2}$,

$$f_I = \frac{32\epsilon^{*J}_{\mu\nu}p_b^\mu p_b^\nu\sqrt{z^3}(1-z+\ln z)}{m_b(1-z)^3}\ln\big(\frac{\lambda^2}{m_b^2}\big) + finite\ terms, \quad (16)$$

$$\begin{aligned}f_{II} = &\frac{8\epsilon^{*J}_{\mu\nu}p_b^\mu p_b^\nu\sqrt{z}}{m_b^3(1-z)^3}\int_0^1 d\xi \frac{\phi_B(\xi)}{\xi}\int_0^1 dy \frac{\phi_K(y)}{\bar{y}^2}\\ &\times[z + (1-z)\bar{y}]. \quad (17)\end{aligned}$$

Here $z = M^2/m_B^2 \approx 4m_c^2/m_b^2$, $\xi$ is the momentum fraction of the spectator quark in the $B$ meson and $y = (1-\bar{y})$ is the momentum fraction of the $s$ quark inside the $K$ meson, $\phi_B$ and $\phi_K$ are the light-cone wave functions for the $B$ and $K$ mesons respectively[2]. Here $\lambda$ is the gluon mass introduced to regularize the infrared divergences in the vertex corrections. We have simplified the results for $f_{II}$ by noting that $\xi \sim \mathcal{O}(\Lambda_{\rm QCD}/m_b) \to 0$.

The integrals for $f_{II}$ in Eqs.(15),(17) will give logarithmic divergences when the asymptotic form $\phi_K(y) = 6y(1-y)$ for the kaon twist-2 light-cone distribution amplitude as well as the same parameterization for the $\xi$ integration as that in Ref.[2] are used. It is also worthwhile to emphasize that the infrared singularities in $f_I$ are more serious than the end-point singularities in $f_{II}$. This is because, when one considers the effects of parton transverse degrees of freedom for $B$ and $K$ mesons, end-point singularities arising from spectator interactions can be regularized [10] and logarithmic divergences can then be removed. However, the infrared divergences due to nonfactorizable vertex corrections still exist.

Since two large scales are involved, we can have several different choices of the heavy quark limit. In the above calculations, we have chosen the heavy quark limit as $m_b \to \infty$ with $m_c/m_b$ fixed. In this limit, the infrared divergences in $f_I$ as well as the logarithmic divergences in $f_{II}$, will break down QCD factorization. Another choice of the heavy quark limit is that $m_b \to \infty$ with $m_c$ fixed. Then all the divergences mentioned above are power corrections and should be dropped out, so QCD factorization still holds in this limit. Physically, the latter case is equivalent to the limit of zero charm quark mass in which charmonium is regarded as a light meson. Obviously, the first choice of the heavy quark limit with $m_c/m_b$ fixed is more interesting in phenomenological analysis, and is also usually used in theoretical studies[2, 11], where it is expected that QCD factorization should apply to $B$ meson exclusive decays into charmonium in the limit $m_c \to \infty$ with corrections of order $\Lambda_{\rm QCD}/(m_c\alpha_s) \sim 1$. Our result shows that this expectation holds for decays to S-wave charmonia but not for P-wave charmonia we studied above where the vertex infrared divergence will break down factorization at order of $\Lambda_{\rm QCD}/(m_c\alpha_s)$.

It is worthwhile to discuss the origin of the infrared divergences and the differences between S-wave and P-wave charmonia as well as the differences among P-wave charmonium states. We write momenta for $c\bar{c}$ as $p_c = p/2 + q$, $p_{\bar{c}} = p/2 - q$. The gluon coupling to the $c\bar{c}$ pair showing in Fig.1 is given by

$$J_\nu = \frac{\gamma_\nu(\not{p}_c + \not{k} + m_c)\Gamma}{(p_c + k)^2 - m_c^2} - \frac{\Gamma(\not{p}_{\bar{c}} + \not{k} - m_c)\gamma_\nu}{(p_{\bar{c}} + k)^2 - m_c^2}, \qquad (18)$$

where $\Gamma$ denotes for the weak decay vertex. When $k$ is soft, the coupling in the nonrelativistic expansion in terms of $q \cdot k / p \cdot k$ can be simplified to

$$J_\nu \approx 4\Gamma\Big[\frac{q_\nu}{p \cdot k} - \frac{(q \cdot k)p_\nu}{(p \cdot k)^2}\Big], \qquad (19)$$

where we used the on-shell conditions for $c$ and $\bar{c}$ quarks. For S-wave charmonium states such as $J/\psi$, since the wave functions depend on $q^2$, $q$ in Eq.(19) makes no contribution to the lowest order. So there is no infrared divergence for S-wave charmonium states. For $\chi_{c1}$, because the orbital and spin momenta couplings are in an anti-symmetric form (see Eq.(8)), when combined with the $B \to K$ form factors the infrared divergences arising from Eq.(19) are totally cancelled out (note that this cancellation only accidentally holds because of the specific form of $B \to K$ form factors). However, for $\chi_{c0}$ and $\chi_{c2}$, because the orbital and spin momenta couplings are in a symmetric form, infrared divergences arising from Eq.(19) are not cancelled out. This qualitative argument is confirmed by our explicit calculations performed above.

It is well known that there are infrared divergences in the *inclusive* decay and production of P-wave charmonia, which are related to the problems encountered here. In the *inclusive* processes, the infrared divergences in the color-singlet P-wave $c\bar{c}$ state can be removed by including contributions from the higher Fock states with color-octet $c\bar{c}$ pair (say in S-wave) and soft gluon within the NRQCD factorization framework (e.g. for B decay see [12], for annihilation hadronic decay see [13, 14], and for decay and production see [15, 16, 17]). However, to the best of our understanding, the color-octet $c\bar{c}$ pair with dynamical soft gluon can make contributions to the multi-body but not two-body *exclusive* decays. (Note that the soft gluon emitted by the color-octet $c\bar{c}$ and subsequently reabsorbed by the spectator light quark has already been considered in the nonfactorizable spectator contribution discussed above (see diagrams (e,f) in Fig.1), where the soft gluon is related to the end-point singularities of $B$ and $K$ light-cone wave functions.) As a result, it seems to us that the infrared divergences encountered in *exclusive* processes involving charmonia may raise a new question to the QCD factorization and NRQCD factorization in B exclusive decays. Further studies are needed to seek the solution to remove the infrared divergences.

In summary, we have studied the exclusive two-body decays of $B$ meson into the spin-triplet P-wave charmonium states $\chi_{cJ}(J = 0, 1, 2)$ and kaon within QCD factorization and NRQCD by adopting the color-singlet nonrelativistic wave function for charmonium. We find that for these decays (except $B \to \chi_{c1}K$), there are infrared divergences arising from nonfactorizable vertex corrections, which can not be removed by including contributions from the color-octet $c\bar{c}$ with soft gluon Fock states, and therefore will break down QCD factorization. New considerations should be introduced to describe $B$ meson exclusive decays to charmonium states.

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